

Stability of zero-mode Landau levels in bilayer graphene against disorder in the presence of the trigonal warping

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The stability of the zero-energy Landau levels in bilayer graphene against the chiral symmetric disorder is examined in the presence of the trigonal warping. Based on the tight-binding lattice model with a bond disorder correlated over several lattice constants, it is shown that among the four Landau levels per spin and per valley, two Landau levels exhibit the anomalous sharpness as in the absence of the trigonal warping, while the other two are broadened, yielding split peaks in the density of states. This can be attributed to the fact that the total chirality in each valley is ± 2 , which is protected topologically even in the presence of an intra-valley scattering due to disorder.

I. INTRODUCTION

In bilayer graphene, the Landau level structure in the vicinity of the contact point is drastically changed by the trigonal warping. In the presence of the trigonal warping, the four Dirac cones appear in low energies out of the parabolic bands touching at K and K' points¹. Due to this modification, we have four-fold zero-energy Landau levels per spin and per valley in weak magnetic fields. So the trigonal warping doubles the degeneracy of the zero-energy Landau levels from the situation in the absence of the trigonal warping. Now an interesting question is how the four Landau levels react to disorder.

Bilayer graphene, in the absence of the trigonal warping, has two-fold degenerated zero-energy Landau levels, and the stability of such zero-modes against disorder in gauge degrees of freedom (ripples) has been shown to be a consequence of the index theorem². We have also demonstrated with an explicit numerical approach based on a tight-binding model as well as with Aharonov-Casher argument that the doubly-degenerated zero-energy Landau levels ($n = 0$ and $n = 1$) exhibit an anomalous sharpness for a bond disorder spatially correlated over more than several lattice constants and respecting the chiral symmetry of the system³. So the question now is whether the additional two zero-energy Landau levels in the presence of the trigonal warping in a weak magnetic field are all robust against the bond disorder as well. It is to be noted that the chiral symmetry itself is respected by the trigonal warping.

To clarify the stability of these zero-energy Landau levels in the presence of the trigonal warping, we have performed numerical calculations with a tight-binding lattice model in weak magnetic fields to examine the robustness against the spatially-correlated bond disorder. We shall find that, among the four Landau levels per spin and per valley, two levels exhibit the anomalous sharpness against the finite-ranged bond disorder as in the absence of the trigonal warping, while the other two are broadened. This can be attributed to the fact that the *total chirality of the four Dirac cones* at K(K') is $+2(-2)$, and that, remarkably enough, these are topologically protected even in the presence of the intra-valley scattering arising from the disorder.

II. MODEL

We adopt a tight-binding lattice model for bilayer graphene with the Bernal (A-B) stacking having the inter-layer couplings γ_1 and γ_3 (Fig.1(a)). The coupling γ_1 connecting the A_2 site in the top layer and the B_1 in the bottom determines the overall parabolic dispersion, while the coupling γ_3 between the B_2 and the A_1 introduces the trigonal warping of the Fermi surface and produces the four Dirac cones at K(K') point in a low-energy scale¹. We introduce the bond disorder δt in the nearest-neighbor hopping t in each layer as in Ref.³. The disorder is assumed to have a gaussian distribution with variance σ and have a spatial correlation as $\langle \delta t(\mathbf{r}) \delta t(\mathbf{r}') \rangle = \sigma^2 \exp(-|\mathbf{r} - \mathbf{r}'|^2 / 4\eta^2)$ with the correlation length η . In order to access to the low-energy regime in weak magnetic fields, we consider a system as large as 4×10^6 sites which is 4 times larger than those in our previous studies^{3,4} to evaluate the density of states from the Green function $\rho = -\text{Im} \langle G_{r,r}(E + i\varepsilon) \rangle_r / \pi^5$. The magnetic field applied perpendicular to the graphene sheet is taken into account by the Peierls phases for the hopping amplitude, where the magnetic flux per hexagon of the honeycomb lattice is denoted by ϕ in units of the flux quantum $\phi_0 = h/e$. Spin degrees of freedom are neglected for simplicity, and all the lengths are measured in units of the nearest-neighbor distance, a , of the honeycomb lattice.

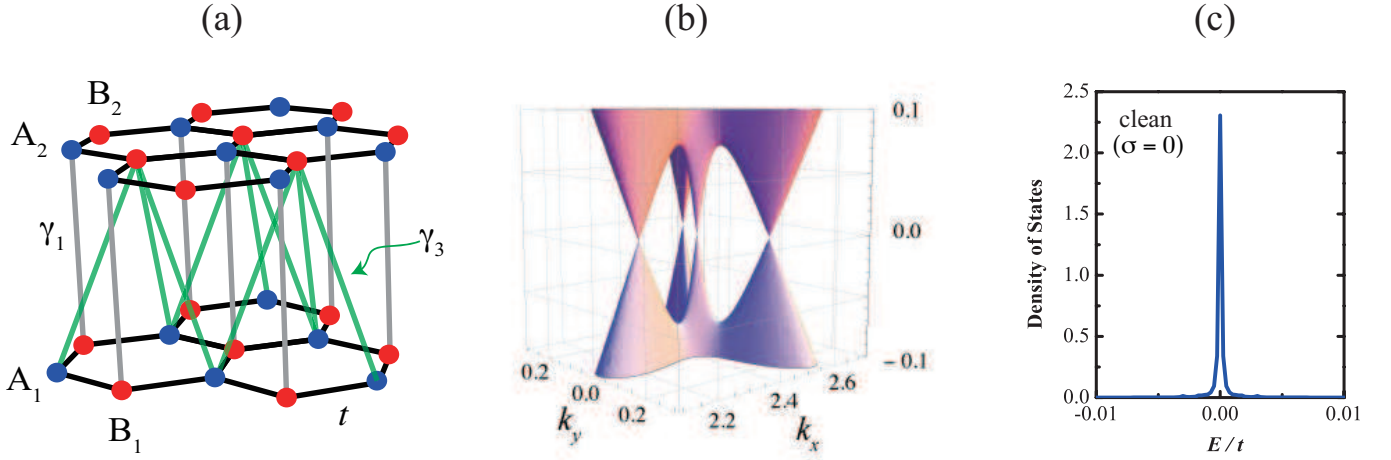


FIG. 1: (a) Lattice model for bilayer graphene with the largest interlayer coupling γ_1 and another, γ_3 , that gives rise to the trigonal warping. (b) Energy dispersion $E(k_x, k_y)/t$ around $(k_x, k_y) = (4\pi/3\sqrt{3}, 0)$ for the interlayer coupling $\gamma_1/t = 0.2$ and $\gamma_3/t = 1.5$. (c) Density of states around $E = 0$ for $\phi/(h/e) = 1/5000$ evaluated by the Green function $G(E + i\varepsilon)$ with $\varepsilon/t = 1.0 \times 10^{-4}$, which can be fit as $(4\phi/\pi\phi_0)(\varepsilon/(E^2 + \varepsilon^2))$.

III. NUMERICAL RESULTS

In order to study numerically the low-energy physics where the trigonal warping (coming from γ_3) is relevant, we assume $\gamma_1/t = 0.2$ and $\gamma_3/t = 1.5$ in the above lattice model. Although the present value of γ_3 is much larger than the value estimated for the bulk graphite $\gamma_3/t \sim 0.1^6$, the dispersion at low energies is still described by the four Dirac cones as shown in Fig.1(b), which should be adiabatically connected to the realistic situation with $\gamma_1/t = 0.12$ and $\gamma_3/t = 0.1$. It is expected therefore that the stability of the zero-mode Landau levels against the disorder scattering, which mixes the four Dirac cones, can be discussed within the present lattice model by assuming an appropriate (see below) value of the correlation length η of the bond disorder.

First, we show in Fig.1(c) the density of states for $\phi/\phi_0 = 1/5000$ in the absence of disorder ($\sigma = 0$), in which we clearly see that the zero-energy Landau level is 4-fold degenerated per valley. It is to be noted that the zero-energy Landau levels in a tight-binding lattice model with a finite system-size have small but finite widths. Exact zero-energy states with the δ -function density of states are realized only in the thermodynamic and continuum limit. In the present calculation, such a small width, however, is much smaller than the energy resolution determined by the imaginary part ε of the energy, since the sharp peak in the absence of disorder can be fitted fairly accurately by the Lorentzian form $\varepsilon/(E^2 + \varepsilon^2)$ with a pre-factor consistent with the four-fold degeneracy of the Landau levels.

We then introduce the disorder in Fig.2. For the case where the bond disorder is uncorrelated in space ($\eta = 0$), we confirm that the usual broadening occurs for the zero-energy Landau levels (Fig.2(a)). In contrast, when the disorder is correlated over a few lattice constants ($\eta/a = 2$), we have one central sharp Landau levels accompanied by two broadened ones (Fig.2(b),(c),(d)). The peak height of the central peak is one-half of that in the absence of disorder (Fig.1(c)), and its shape is insensitive to the disorder strength σ . The splitting of the broadened satellite peaks becomes larger for stronger disorder. The satellite peaks indicate the splitting of the critical energy for the quantum Hall transition, which has been discussed for the monolayer graphene with a short-ranged bond disorder⁷.

All these results suggest that among four-fold zero energy Landau levels, two Landau levels remain anomalously sharp against the spatially correlated bond disorder as in the absence of the trigonal warping ($\gamma_3 = 0$), while the other two are broadened and split as two satellite peaks. If one notes that the separation Δk between Dirac cones for the present model with $\gamma_3/t = 1.5$ and $\gamma_1/t = 0.2$ is order of $0.2a^{-1}$ (Fig.1(b)), one might expect that there should be a considerable disorder scattering among these Dirac cones, since the correlation length $\eta/a = 2$ of disorder is smaller than $\Delta k^{-1} \sim 5a$. In actual bilayer graphene, where γ_3 is much smaller, the separation Δk is estimated to be $\Delta k = 2\gamma_1\gamma_3/(3t^2a)^{1/2}$, which gives $\Delta k^{-1} \sim 120a \sim 17\text{nm}$ for the bulk values of γ_1 and γ_3 . The scale of ripples, on the other hand, is estimated to be $10 \sim 15\text{nm}^{8,9}$, which is again smaller than $\Delta k^{-1} \sim 17\text{nm}$. We thus expect that the mixing of the four Dirac cones due to ripples can be also relevant in actual bilayer graphene.

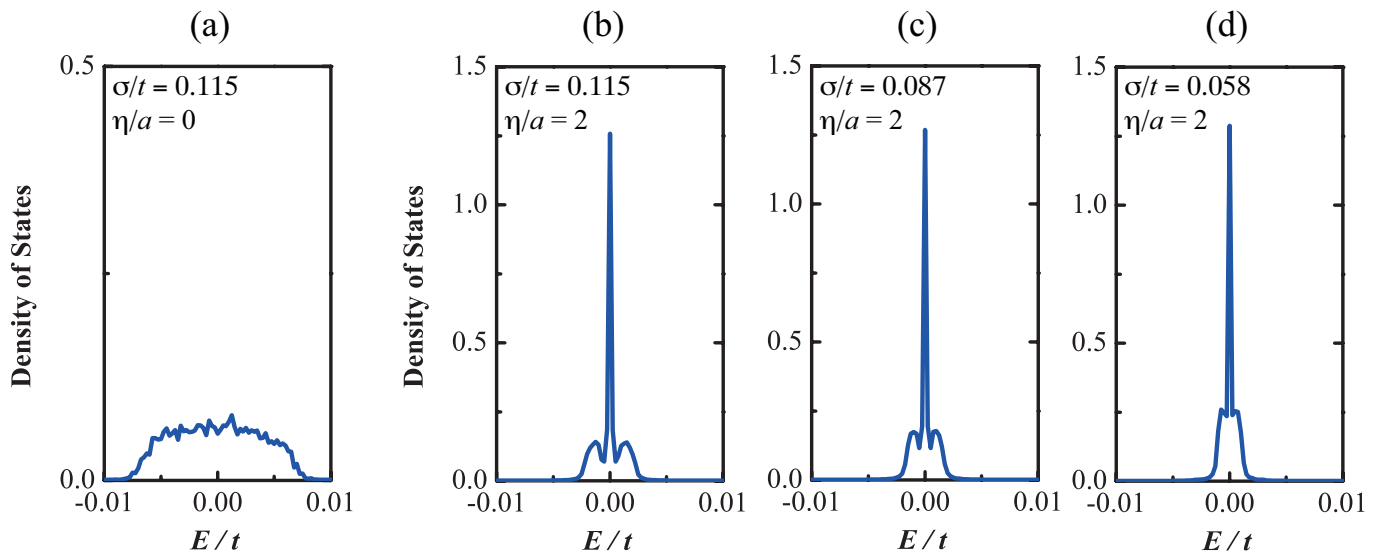


FIG. 2: Density of states at zero energy for an uncorrelated disorder with $\eta = 0$ (a) and for a correlated disorder with $\eta/a = 2$ ((b),(c),(d)). Disorder strength is assumed to be $\sigma/t = 0.115$ for (a) and (b), while $\sigma/t = 0.087$ for (c) and $\sigma/t = 0.058$ for (d). The magnetic flux per hexagon is $\phi/\phi_0 = 1/5000$ and the imaginary part of energy is $\varepsilon/t = 1.0 \times 10^{-4}$.

IV. SUMMARY AND DISCUSSIONS

We have investigated the stability of zero-mode Landau levels of bilayer graphene in small magnetic fields where the trigonal warping is relevant. We have considered a bond disorder that respects the chiral symmetry and is correlated over a few lattice constants, which suppresses the inter-valley scattering but causes the intra-valley scattering. We have found that, among four-fold Landau levels per spin and per valley, two Landau levels are stable against such a disorder with the anomalously sharp density of states at $E = 0$. On the other hand, the other two levels are broadened and yield split satellite peaks.

This result can be attributed to the fact that the total chirality of the four Dirac cones at K (K') is $2(-2)$, which is topologically protected even in the presence of intra-valley scattering due to disorder. When the inter-valley scattering is switched on by making the disorder uncorrelated spatially ($\eta = 0$), the anomalously sharp peak is washed away (Fig. 2(a)). The present results then suggest that the mixing of the two Dirac cones with opposite chirality by the disorder scattering generally destroys the anomalous stability of zero-mode Landau levels of each Dirac cone.

Acknowledgments

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